Title: Variable Expressions in Algebra

Brief Overview:

Algebra could never exist without the concept of variables. Variables and algebra are synonymous. The concept of variable expression, therefore, is the rudiments of algebra, that is, it is the fundamental of algebra.

These lessons are the first three lessons when introducing algebra. When students know that variables can be used to represent quantities, then their understanding of the concepts of algebra begins.

By the end of these three lessons the students should have a fundamental perception of what algebra is all about. The student will also be able to perform some of the "arithmetic" operations in algebra.

NCTM Content Standard/National Science Education Standard:

- NCTM standard 1.8.2 - simplify expression by combining like terms and applying order of operations.
- Use mathematical properties to justify the steps in simplifying algebraic expressions.

Grade/Level:

Grade 7

Duration/Length:

Three lessons - 60 minutes each.

Student Outcomes:

Students will:

- Identify algebraic expressions.
- Distinguish a coefficient from a constant.
- Identify variables.
- List terms in an algebraic expression.
- Identify like terms.
- Simplify expressions by combining like terms.
- Simplify variable expressions by using the multiplicative and distributive properties.

Materials and Resources:

Pencil and Paper

Development/Procedures:

Lesson 1

Preassessment – Students should be able to effectively solve problems using the four basic operations of mathematics.

24 + 38 = ?	**Answer = 62**
42 + 38 + 57 = ?	**Answer = 137**
32 + 78 – 29 = ?	**Answer = 81**
43 – 58 + 17 = ?	**Answer = 2**
2 + 21 ÷ 3 x 5 = ?	**Answer = 37**
2 + 21 ÷ 3 – 2 x 4	**Answer = 1**
72 ÷ (3 + 5) + 5	**Answer = 14**

Launch –

Terri has 3 boxes of apples 2 boxes of bananas and 4 boxes of cups. Is it appropriate to ask how many total items Terri has? (This is a discussion question leading to the concept that you can't mix apples with bananas or cups.)

Joan has a box of apples and some apples outside the box. Is it appropriate to ask how many apples in total that Joan has? This is again a discussion question leading to the concept that you can add an unknown number of apples to a known number of apples. Thus leading to the correct question: What is the total number of apples that Joan has?

Is it possible to add 8 pencils to 7 books?

Teacher Facilitation – Terri was unable to add apples to bananas or even to cups. Apples, bananas and cups are three different terms. Suppose we use 'a' for number of apples, 'b' for number of bananas and 'c' for number of cups. a, b, and c are called **variables**. Variables are symbols or letters that can be used to represent numbers.

We can now represent what Terri has as 3a + 2b + 4c. In the expression 3a + 2b + 4c, each addend is know as a term. 3a + 2b + 4c has 3 terms. 3a, 2b, and 4c are each known as **terms**. Suppose Terri has 5 dollars in her pocket. She decides to write all the things she has as 3a + 2b + 4c + 5. This has four terms
a, b, c are **variables**
3, 2, 4 are **coefficients**

5 is a **constant. Constants do not have variables attached to them while coefficients do.**
3a is an **algebraic expression**
4c + 5 is an **algebraic expression**
3a + 2b + 4c + 5 is an **algebraic expression**

An **algebraic expression** is an expression that contains at least one term. Other algebraic expression can be in the form of 3a ÷2; 7xy × 5; 8yz - 2y; etc.

Joan has a box of apples and some single apples. How can you add these two? We don't know how many apples we have in the box. If we represent a single apple as 'a', we cannot represent a box of apples with 'a' because a box of apples has more than one apple in it. We may then represent what Joan has as A + a where A represents a box of apples and 'a' a single apple.

How can you write the third statement as an algebraic expression?
Answer =A + a
How many terms does it have? **Answer = 2**
What are the terms? **Answer=A, a**
Which are the variables? **Answer= A, a**
Write down the coefficients.**Answer=1, 1**
What is the constant?**Answer= 0**

Student Application

1. What are the terms of
   - **a.** 3a + 5b – 7c **Answer=3a, 5b, -7c**
   - b. 8x – 10y – 20z – 1 **Answer= 8x, -10y, -20z, -1**
   - **c.** 10r – 28q + 5 **Answer=10r, -28q, 5**
2. Write the coefficients in Question 1 above.
   - **a. Answer=3,, 5, -7**
   - **b. Answer=8, -10, 20**
   - **c. Answer=10, -28**
3. Write the constants in Question 1 above.
   - **a. Answer=0**
   - **b. Answer=–1**
   - **c. Answer=5**

Embedded Assessment : Students correctly identify algebraic terms.

Re-teaching/Extension –
For students who have not fully understood the concept, reinforce by re-teaching with easier questions.

1. Name the coefficients of the variable terms.
   a. 5x + 3 **Answer=5**
   b. 3xy **Answer=3**
   c. –3n **Answer=–3**
   d. 8x – 3y **Answer= 8, -3**

2. Name the constant term in Question 1 above.
    a. **Answer=3**
    b. **Answer=0**
    c. **Answer=0**
    d. **Answer=0**
3. How many terms are in each of these variable expressions in Question 1 above?
    a. **Answer=2**
    b. **Answer=1**
    c. **Answer=2**
    d. **Answer=2**

For students who have understood the concept, I will give them harder problems.

1. Name the coefficients of the variable terms.
    a. $3x^2 + 5x + 3$ **Answer=3, 5**
    b. $6x^2y + 3xy^2 + 8$ **Answer=6,3**

2. Name the constant term in these variable expressions.
    a. $-2n^2 - 3n + 7$ **Answer=7**
    b. $3x^2 + 4x - 6$ **Answer=-6**
    c. $x^2 - 8x + 3$ **Answer= 3**

3. How many terms are in each of these variable expressions?
    a. $4x^2 - 2y + 3xy + x - 8$ **Answer=5**
    b. $6a^2 - 4ab - b^2$ **Answer=3**
    c. $y^2 - 4y^2 - y + 9$ **Answer=4**
    d. $8 - a^3$ **Answer=2**

4. Name the variable terms of the expression.
    a. $5xy^2 + 3xy^2 + 5$ **Answer= $5xy^2$, $3xy^2$**
    b. $p^3 - 4p^2 - p + 9$ **Answer= p^3, $-4p^2$, $-p$**
    c. $5 - 5q + 2q^2$ **Answer= $5q$, $2q^2$**

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Lesson 2
Preassessment

Students should be able to identify algebraic expressions. They should be able to distinguish a coefficient from a constant, as well as identify a variable. They should equally be able to evaluate variable expressions.

The following class exercises are given to ascertain the students' understanding of the concept taught.

1. John traveled to a supermarket in Riverdale and bought 5 cartons of soda. He went to another supermarket in Hyattsville and bought 4 cartons of soda. How

many cans of soda did he buy altogether? **Answer= 9 c. Let c = number of cans in each carton. 5c+4c=9c.**

2. On getting home, he gave 2 cartons to an important guest as a gift. How many cans does he have left? **Answer= 7 c**
3. Janet has 5 boxes of doughnuts, 7 boxes of pencils, and 2 packets of rulers.
    a. What is the total sum of all the items? **Answer= 5d + 7p + 2r**
    b. How many variables are there? **Answer= 3**
    c. Give the coefficient of each variable. **Answer= 5, 7, 2**
    d. What is the constant of this algebraic expression. **Answer=0**

Launch

Keri bought 8 boxes of doughnuts from shop A and 3 boxes of doughnuts from shop B. How many doughnuts did she buy all together? **Answer=11d. Remember d=number of doughnuts in each box.**

Can you solve this problem? Yes. What is the answer?

Keri gave 2 boxes of her doughnuts to her favorite teacher, how many doughnuts does she have then? **Answer= 9 d**

The teacher has 2 boxes of doughnuts and 2 boxes of cups of coffee. What is the sum of what he has?

Answer=2d +2c

Teacher Facilitation

We are able to get the solution to the first question is 11 d.

You can decide to use 'd' to represent doughnuts.

Then $8d + 3d = (8 + 3)d = 11d$

Similarly, the second question can be expressed as $11d - 2d = (11-2)d = 9d$

But the third question which can be written in short as $2d + 2c$ is difficult to write as one term. This is because the two items are different.

When we have an algebraic expression, we can easily perform any of the four basic operations on terms that are alike.

For example: evaluate $2x + 3y - x + y$

First bring together the terms that are alike.

$$= 2x - x + 3y + y$$
$$= x + 4y$$

Another example, evaluate $4xy^2 + 3x + 5xy^2 + 5x$

Bring the like terms together.

$$= 3x + 5x + 4xy^2 + 5xy^2$$
$$= 8x + 9xy^2$$

Here we have simplified our expressions. To simply in this case is to collect like terms.

Algebraic expressions can also be evaluated by substituting the variables with positive or negative numbers.

For example: Evaluate the variable expression when a = 3, b = 2, and c = 1.

i. $4a + 3b$
$(4 \times 3) + (3 \times 2)$
$= 12 + 6$
$=18$

ii $-4c + 4$
$(-4 \times 1) + 4$
$-4 + 4$
0

iii $3a - (c + a)^3$
$(3 \times 3) - (2 + 3)^3$
$(3 \times 3) - (5)^3$
$(3x3) - (5 \times 5 \times 5)$
$9 - 125$
-116

Student Application

1. Simplify $4x - 3y + 2x + 10y$ **Answer = 6x + 7y**
2. $7x^2 + 3y^2 + 4x^2 - y^2$ **Answer = $11x^2 + 2y^2$**
3. Evaluate $4a + 5b$ where a = 3, b = -2 **Answer = 2**
4. Evaluate the expression $3a - (a + b)^2$ where a =2, b =3 **Answer = -19**
5. Evaluate $\frac{c - 2a}{ac^2 - d}$ where a =2, c =3, d= 4 **Answer = -1/14**

Embedded assessment: Students simplify algebraic expressions correctly.

Re-teaching/Extension

If the students do not understand the concept, I will offer them the following simpler exercises.

1. Evaluate the variable expressions when a = 3, b = 4, c = -2
   - **i.** $b - 4a$ **Answer= -8**
   - ii. $c^2 - a^2$ **Answer= -5**
   - iii. $\frac{a^2 - c}{abc}$ **Answer = -11/24**
   - iv. $\frac{b-a}{c}$ **Answer = -1/2**
   - **v.** $b^3 - 3ab + c$ **Answer = 26**

For the students who understand the concept, ask them to make up their own problems and have them give these to their classmates to solve.

Lesson 3

Preassessment – Evaluate (i) $5x - 3y - z$ where $x=2$, $y=3$, $z=1$

Answer 0

(ii) $5x- (3y-z)$ **Answer =2**

2. Simplify (i) $3x - 5x^2 + 2x + x^2$ **Answer $5x-4x^2$**

(ii) $17 - (2a+5) + 4a$ **Answer= 12+2a**

Launch : (a). Angie has 2 pencils, Betty has double what Angie has, how many pencils does Betty have? **Answer 4 pencils**

(b) Chris has three times as many pencils as Betty. How many pencils does Chris have? **Answer 12 pencils**

(c) Duane has twice as many pencils as Angie and Betty. How many pencils does Duane have? **Answer 12 pencils**

Teacher Facilitation:

(a) Angie has 2 pencils. Betty has 2x2 pencils = (2x2)pencils
= 4 pencils

(b) Chris has 3 times as many pencils as Betty = (3x4) pencils
= 12 pencils
or (3x2x2) pencils
= (3x2) x2 pencils
= 6x2 pencils
= 12 pencils

(c) Duane has 2 (2+4) pencils
= 2(6) pencils
= 12 pencils
or 2(2+4) = (2x2) + (2x4) pencils
= 4+8 pencils
= 12 pencils

Note: In Chris's case he has (3x4) pencils
= (4x3) pencils
= 12 pencils

This is the **order** property which otherwise is known as the **communicative** property.

Chris's problem could also be solved as (3x2)x2 **or** 3(2x2)

6x2 **or** 3x4
12 **or** 12

(3x2)x2= 3(2x2) this is the **group** property which otherwise is known as the **associative** property. The number and the operations are kept in exactly the same order. The only change is the grouping, that is, the way that the parentheses are inserted.

Duane's case is a bit different. He has 2(2+4) = (2x2) + (2x4)

2x6 **or** 4+8
12 **or** 12

This is the Distributive Law of Multiplication over Addition.
Simplify:

1. 3(2x + 3y)
   3(2x) + 3(3y)
   6x + 9y
2. 3(2x + 3y) + x
   3(2x) + 3(3y) + x
   6x + 9y +x
   6x +x +9y
   7x +9y
3. 5(a +2b) – 3(b-a)
   5a +10b – 3b +3a
   5a+3a+10b-3b
   8a+7b
4. $7p-2(p^2-2p)+3p^2$
   $7p-2p^2+4p+3p^2$

$7p+4p+3p^2-2p^2$
$11p + p^2$

Student Application:

1. Name the following properties being used:
   a. 3x+4y=4y+3x **Answer communicative**
   b. 3x+(4y+5z)+(3x+4y)+5z **Answer Associative**
   c. 3(x+y)=3x+3y **Answer distributive**
2. Simplify:
   a. $7(x-2y)+8(2x+y)$ **Answer =23x+6y**
   b. $3x-5z-3(x-5z)$ **Answer = 10z**
   c. $10(p^2-q)+10(p^2+q^2)$ **Answer=**
   $20p^2-10q+10q^2$

Embedded Assessment : The embedded assessment is the students solving the problems and correctly identifying the properties being used..

Re-teaching/Extension: The students will make up their own quizzes based on the properties learned for the teacher to solve.

Summative Assessment:

Take home quiz,

1. Write three questions about your friends to demonstrate clearly your understanding of the commutative property.
2. Look around your room, using items in your room, write three questions that require the use of the associative property.
3. Pretend you are shopping with your mother. Make up three questions based on the items you bought. Use your knowledge of the distributive property to calculate the amount of money mom spent.

*** Use variables to denote your items.

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